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Joint Research Institute: Signal and Image Processing

ML Estimation for Unknown Numbers of Signals

Pei-Jung Chung

Problem Formulation

An array of *n* sensors receives *m* narrow band far field signals impinging from unknown directions

$$\underline{\theta}_m = [\theta_1, \cdots, \theta_m]^T$$

hidden in additive Gaussian noise (spatially and temporally white). The array output is given by

$$\underline{x}(t) = \underline{\underline{H}}(\underline{\theta}_m)\underline{s}(t) + \underline{n}(t).$$

- Given the array outputs $\{\underline{x}(t)\}_{t=1}^T$ and a pre-specified number of signals *m*, the ML estimate is obtained from minimizing the negative concentrated likelihood function:

$$\hat{\underline{\theta}}_m = \arg\min L(\theta_m)$$

Robust ML Estimation for Unknown Numbers of Signals

- **Motivation** When the number of signals is unknown, conventional methods compute the ML for a sequence of nested models

$$M_1 \subset M_2 \subset \cdots\cdots M_M$$

and then select the best model order and estimate according to the underlying criterion. The computational cost can be very high due to the multi-dimensional search for each candidate model.

- To improve computational efficiency and robustness against model order uncertainty, our procedure computes the ML for the maximal possible number of signals *M (>=m).*

- The resulting parameter vector

$$\hat{\underline{\theta}}_M = [\hat{\theta}_1, \cdots, \hat{\theta}_i \cdots, \hat{\theta}_M]^T$$

Contains relevant components associated with the true parameters $\underline{\theta}_0$ and redundant parameters that do not correspond to $\underline{\theta}_0$.

- In previous works, we suggested to select relevant estimates according to the increase of likelihood function. The increase in likelihood function is asymptotically zero in the case of redundant components. Here we consider a statistically justified selection procedure to enhance robustness of this algorithm.

Selection of Relevant Components

To identify the relevant components, we consider the following hypothesis test:

Hi : $\underline{x}(t) = \underline{\underline{H}}_{M-1}(\tilde{\underline{\theta}}_{M-1})\tilde{\underline{s}}_{M-1}(t) + \underline{n}(t)$

Ai : $\underline{x}(t) = \underline{\underline{H}}_M(\underline{\theta}_M)\underline{s}_M(t) + \underline{n}(t)$

where $\tilde{\underline{\theta}}_M = [\hat{\theta}_1, \cdots, \hat{\theta}_i, \hat{\theta}_{i+1} \cdots, \hat{\theta}_M]^T.$

- The likelihood ratio leads to the test statistic

$$F_i = \frac{n_2}{n_1} \frac{tr[(\underline{\underline{P}}_M(\hat{\underline{\theta}}_M) - \underline{\underline{P}}_{M-1}(\tilde{\underline{\theta}}_i))\hat{\underline{\underline{R}}}]}{tr[(I - \underline{\underline{P}}_M(\hat{\underline{\theta}}_M))\hat{\underline{\underline{R}}}]}$$

where $\hat{\underline{\underline{R}}}$ is the sample covariance matrix and $\underline{\underline{P}}_m(\hat{\underline{\theta}}_m)$ is the projection matrix onto the column space of $\underline{\underline{H}}_m(\hat{\underline{\theta}}_m)$.

Summary

Input: $\{\underline{x}(t)\}_{t=1}^T, M, t_\alpha$

1. Find the ML estimate $\hat{\underline{\theta}}_M$
2. Compute the statistic
   $F_i, i = 1, \cdots\cdots M.$
3. Test *Hi* against *Ai* to select relevant components *(i=1,...,M):*
   $\hat{\theta}_i$ is relevant if $F_i \geq t_\alpha.$

Output: $\hat{\underline{\theta}}_0 = [\hat{\theta}_{(1)}, \hat{\theta}_{(2)} \cdots, \hat{\theta}_{(k)}]^T$

Conclusion

- The proposed algorithm computes ML estimates only for the maximal hypothesized number of signals and selects the relevant components associated with the true parameters.
- Compared to traditional methods, this approach avoids the full search process through a series of nested models. It leads to significant improvement in computational efficiency and provides comparable estimation accuracy as standard methods.

Numerical Results

Experiment True number of signals: m_0=2,

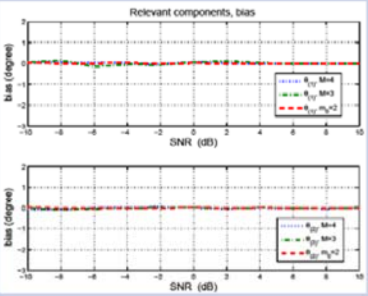
$\underline{\theta}_0 = [28°36°]^T.$

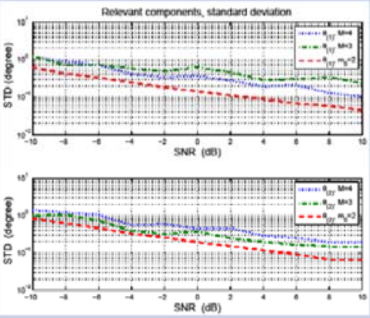
Assumed number of signals M=3, M=4

T=100, SNR = -10 to 10 dB

n=10. SNR_diff=[1 0] dB.

- The bias and standard deviation are plotted for each DOA parameter.
- The bias is close to zero for both *M=3,4*. The variance is increased by the introduction of redundant parameters.





References

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