The origin and non-universality of the earthquake inter-event time distribution

S.Touati, M.Naylor, and I.G.Main

School of GeoSciences, University of Edinburgh, Scotland, EH9 3JW

EPSRC Engineering and Physical Sciences Research Council

ERPem Edinburgh Research Partnership in Engineering and Mathematics

ABSTRACT

Many authors have modelled regional earthquake inter-event times using a gamma distribution, whereby data collapse occurs under a simple rescaling of the data from different regions or time periods. We show, using earthquake data and simulations, that the distribution is fundamentally a non-universal, bimodal mixture distribution dominated by correlated aftershocks at short waiting times and independent events at longer times. The much-discussed power-law segment arises under some circumstances as a crossover between these two.

1. Real earthquake data

Over the past decade, much has been written about the distribution of waiting times between earthquake events. It is common to fit the empirical histograms to a gamma distribution, which suggests universality (Bak et al. 2002), whereby rescaling with the mean event rate produces data collapse onto a universal gamma distribution.

Inter-event time histograms for the global PDE and southern California earthquake catalogues are shown in Fig 1. In the left hand figures it is clear that the regional distribution is in fact bimodal, having two peaks (Fig. 1(a)). More commonly, inter-event time distributions are plotted in the form of the right hand figures where each frequency has been normalised by the bin width, which tends to smooth out the two bumps (Fig. 1(b)). The global data by contrast appears to be unimodal and Poisson-shaped (Fig. 1(c)).

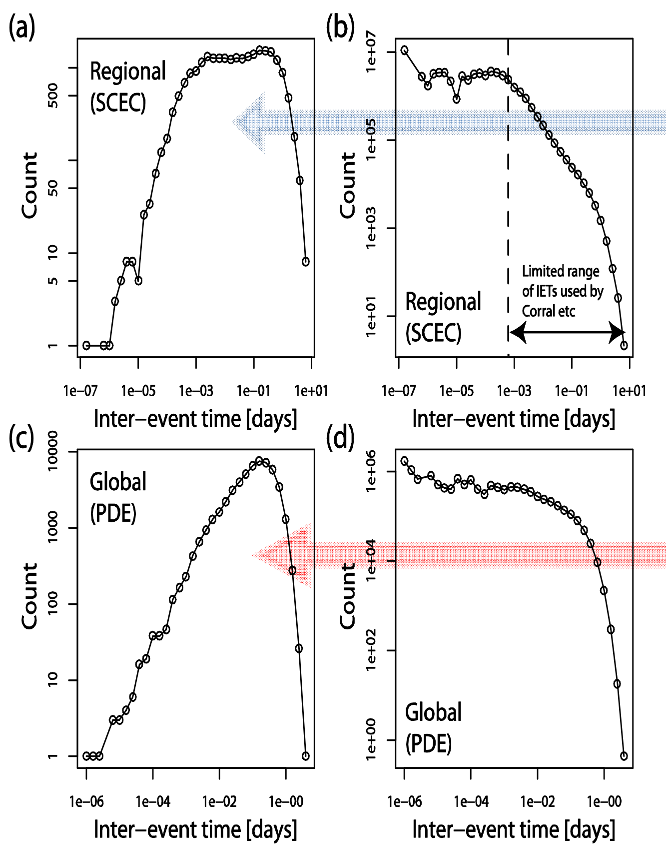


Figure 1 (left): Regional (a,b) and global (c,d) distributions of inter-earthquake times. In the right-hand figures (b,d), the counts have been normalised by the logarithmic bin widths, which disguises the bimodality of the regional distribution.

2. ETAS model simulations

We can recreate the observed distribution shapes using synthetic catalogues generated by the ETAS model (see Box 1). To make comparisons with global and regional earthquake data, we concentrate on the rate of independent events, μ. This parameter can be considered as a proxy for region size (see figure in Box 1).

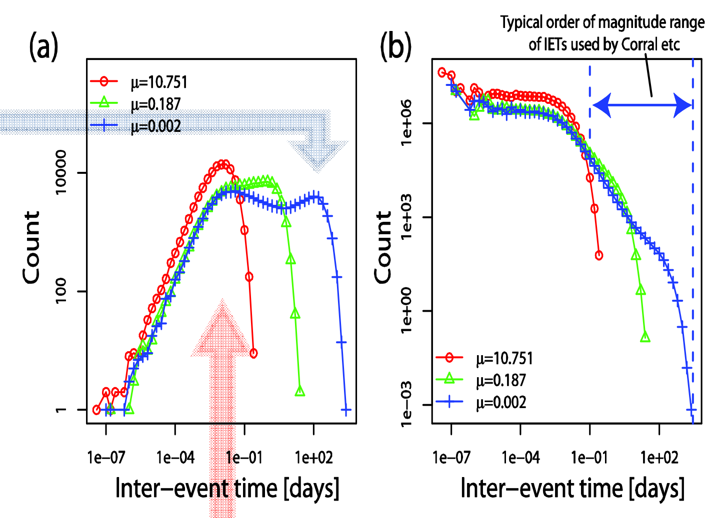


Figure 2 (above): Inter-event time distributions from ETAS simulations, using three different values of the seeding rate μ to mimic the effect of changing the region size. Note the similarity to the distributions in figure 1. The plots in the right-hand figure (b) have counts normalised by the logarithmic bin widths.

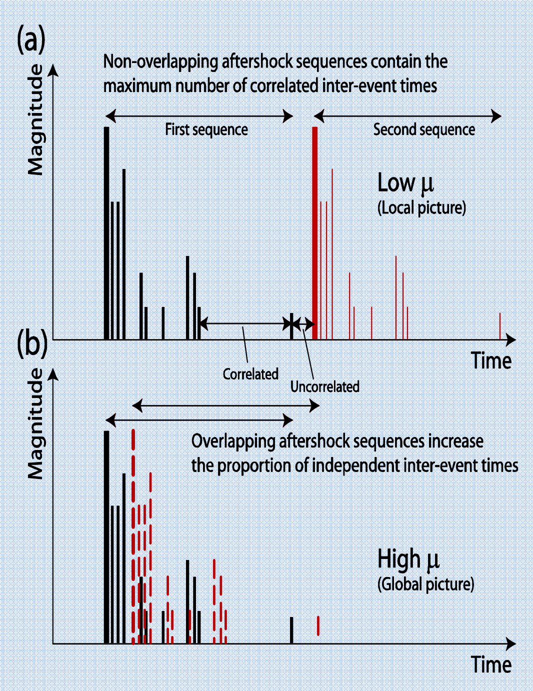
The effect of μ on the synthetic ETAS inter-event time distribution is shown in Fig. 2. For large μ, the shape of the distribution is Poisson. For intermediate values of μ, the peak of the distribution flattens out and we see the familiar gamma distribution. When μ is made very small, however, the straight segment becomes peaked in two places (Fig. 2(a)). Thus as we decrease μ we progress from a unimodal to a bimodal distribution, much like the progression from global to regional scale in real data.

Box 1: Epidemic-Type Aftershock Sequences (ETAS) model

ETAS (Ogata 1988) is a stochastic point process model for earthquake temporal occurrence, incorporating well-known empirical laws of seismicity. The event rate at any instant is given by

$$\lambda(t \mid H_t) = \mu + K \sum_{i:t_i<t} \frac{e^{\alpha(m_i - m_0)}}{(t - t_i + c)^p}$$

Independent "seed" events occur as a Poisson process of rate μ. All events are capable of triggering their own aftershocks. The seeding rate μ therefore determines the rate at which aftershock sequences are initiated (see figure to right), and hence, the extent of their overlapping in the time series. This clearly affects the presence or otherwise of correlations in the inter-event time sequence.



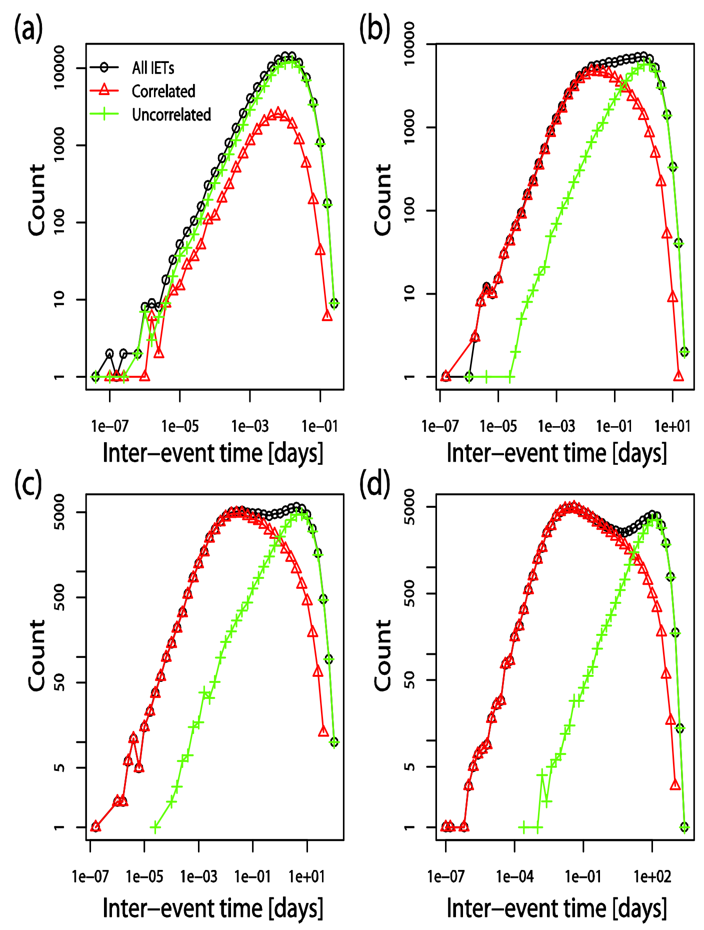


Figure 3 (right): ETAS inter-event time distributions for different μ values, separated into correlated (red) and uncorrelated (green) sub-distributions. This demonstrates the origin of the bimodality in the overall distribution: it is a mixture distribution based on aggregating these two simpler shapes.

3. Origin of the bimodality

The ETAS model allows us to perform further analysis: we may categorize each inter-event time as either correlated, if occurring between two events belonging to the same aftershock sequence, or otherwise, uncorrelated (see figure in Box 1). Fig. 3 shows these correlated and uncorrelated subsets superimposed onto the histograms, for various different values of μ. It is clear that the two sub-distributions have much simpler forms. The uncorrelated waiting times are Poisson-distributed, as expected for independent events. The distribution of correlated waiting times, shown most clearly in Fig. 3(d), is a gamma distribution.

To conclude, we have shown that the distribution of inter-earthquake times is generally bimodal, arising from a mixture of gamma-distributed correlated intervals and Poisson-distributed uncorrelated ones. We therefore reject the hypothesis of universality.

References

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•Ogata, Statistical models for earthquake occurrences and residual analysis for point processes, J. Am. Stat. Assoc. 83(401), 1988