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A Novel Method for Interfacial Stress Analysis of Plated Beams

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Introduction

Recent developments in structural engineering has demonstrated the effective enhancement in strength and performance of reinforced concrete and metallic beams bonded with a thin fibre reinforced polymer (FRP) composite or steel plate on its tension face (Fig. 1). This technique is now popularly adopted for retrofitting existing structures. Interfacial shear and normal stresses are developed between the adherents in such plated beams during the transfer of stresses between the bonded plate and the original beam. The combination of these stresses are responsible for the common premature debonding failure of the plate from the original beam in a brittle manner.

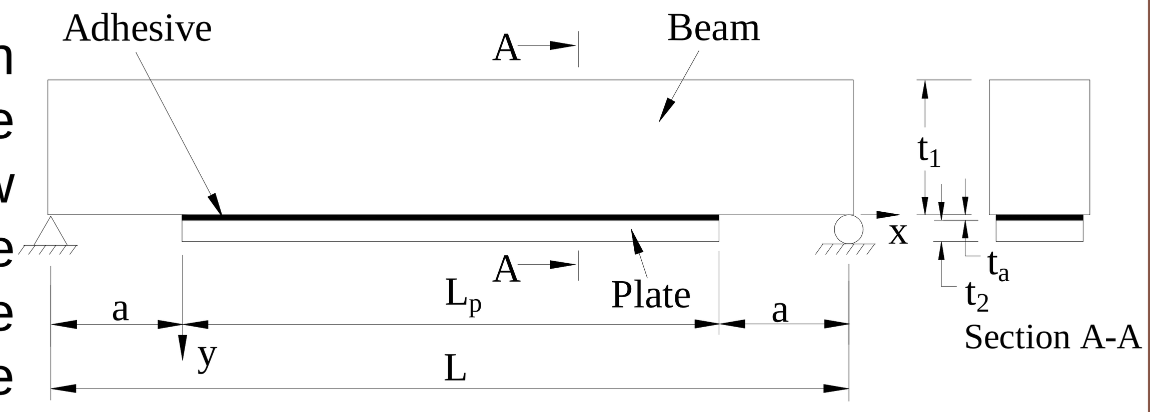

Fig.1 Plated beam

Consequently, many analytical solutions have been developed to quantify these interfacial stresses. However, almost all these solutions are specific to pre-defined simple loading arrangements, so each solution is only applicable to a specific loading. This research presents a new analytical solution for the interfacial stresses in simply supported beams bonded with a tension face thin plate. The solution is generic and applicable to beams and plates made of any materials within the linear elastic range which is common to almost all previous studies. The novelty of this work lies in the application of the superposition principle so that the simple solution is applicable to any arbitrary loading arrangement.

Methodology

The plated beam under an arbitrary loading as shown in Case-1 is split into Case-2 and Case-3. Case-2 includes all the external loading plus an axial force and bending moment at each end of the plate. The magnitude of these axial forces and moments are determined from the deformation of the un-plated beam so that both ends of the plate deform compatibly with the un-plated beam under the external loading and the case can be analysed using the classical composite beam analysis. Case-3 is the plated beam under the same but opposite plate end loading as in Case-2. Case-3 is further decomposed into a symmetrical loading Case-4 and anti-symmetric loading Case-5 within the plated region. The combined solution to Cases 2, 4 and 5 gives the solution for the original problem Case-1.

$$N_{pl} = \frac{M(0)}{E_1 I_1}(y_1 + y_2 + t_a)E_2 A_2$$

$$N_{pr} = \frac{M(L_p)}{E_1 I_1}(y_1 + y_2 + t_a)E_2 A_2$$

$$M_{pl} = \frac{M(0)}{E_1 I_1}E_2 I_2 \; ; \quad M_{pr} = \frac{M(L_p)}{E_1 I_1}E_2 I_2$$

$$N_{ps} = \frac{N_{pl} + N_{pr}}{2} \; ; \quad N_{pu} = \frac{N_{pl} - N_{pr}}{2}$$

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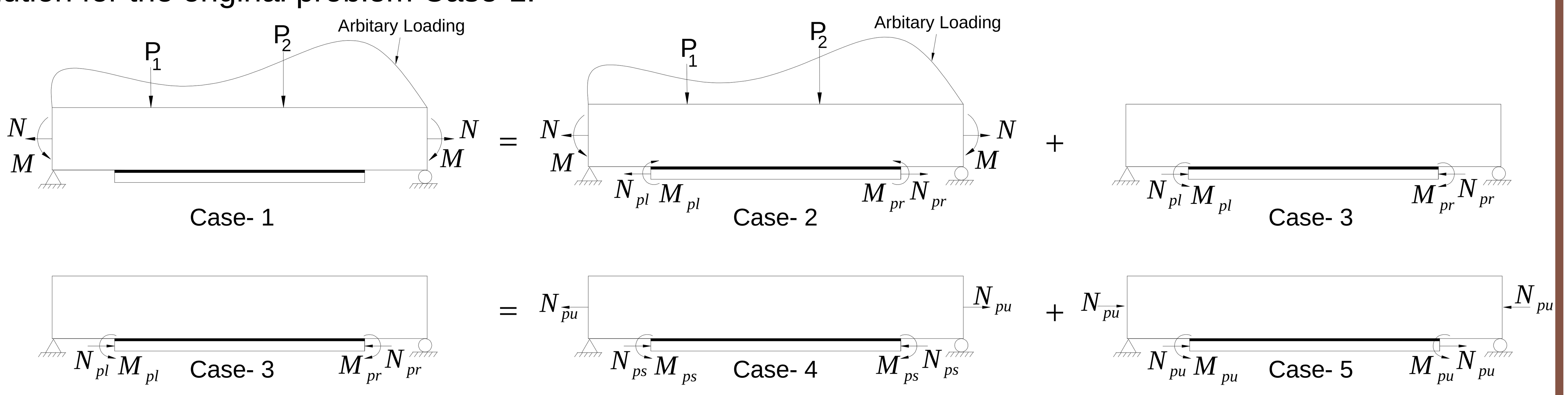


Solution to Case- 1 = Solution from Case- 2 + Case- 4 + Case- 5

Subscripts *l* & *r* refer respectively to the left and right plate ends. Subscripts *s* & *u* refer respectively to the symmetrical and anti-symmetrical loading cases. $M(0)$ and $M(L_p)$ denote moment at $x=0$ and $x=L_p$ respectively on the beam under the original loading. E, A, I, and t refer to the elastic modulus, cross sectional area, second moment of area, and thickness respectively. y_1 & y_2 refer to distance from bottom of beam and top of plate to their respective centroids. Subscripts *1*, *a* & *2* respectively refer to beam, adhesive and plate.

Solution

1. Composite Beam (Case - 2)

Interfacial shear stress

$$\tau(x) = m_c(y)V_{Tc}(x)$$

where $m_c(y) = \frac{Q_e(y)}{I_e b_2}$

Interfacial normal stress

$$\sigma(x) \approx 0$$

2. Symmetrical Loading at Plate Ends (Case - 4)

$$\tau(x) = B_{1s}\left[\cosh(\lambda x) - \sinh(\lambda x)\right]$$

where $B_{1s} = \frac{G_a}{E_1 \lambda}\left(-r_2 M_{ps} + r_3 N_{pu} + r_4 N_{ps}\right)$

$$\sigma(x) = e^{(-\beta x)}\left[C_{1s}\cos(\beta x) + C_{2s}\sin(\beta x)\right] - B_{1s} n_1 \lambda\left[\sinh(\lambda x) - \cosh(\lambda x)\right]$$

where $C_{1s} = \frac{E_a}{2\beta^2 t_a E_2 I_2}M_{ps} - \frac{n_2}{2\beta^3}\tau(0) + \frac{B_{1s} n_1 \lambda^3}{2\beta^3}(\lambda - \beta)$

$$C_{2s} = \frac{B_{1s} n_1 \lambda^3}{2\beta^2} - \frac{E_a}{2\beta^2 t_a E_2 I_2}M_{ps}$$

3. Anti-symmetrical Loading at Plate Ends (Case - 5)

$$\tau(x) = B_{1u}\left[\cosh(\lambda x) - \sinh(\lambda x)\right] + m_1 V_{Tu}$$

where $B_{1u} = \frac{G_a}{E_1 \lambda}\left(r_1 M_{1u} - r_2 M_{pu} + (r_3 + r_4)N_{pu}\right)$

$$\sigma(x) = e^{(-\beta x)}\left[C_{1u}\cos(\beta x) + C_{2u}\sin(\beta x)\right] - B_{1u} n_1 \lambda\left[\sinh(\lambda x) - \cosh(\lambda x)\right]$$

where $C_{1u} = -C_{2u}\tan\left(0.5\beta L_p\right)$

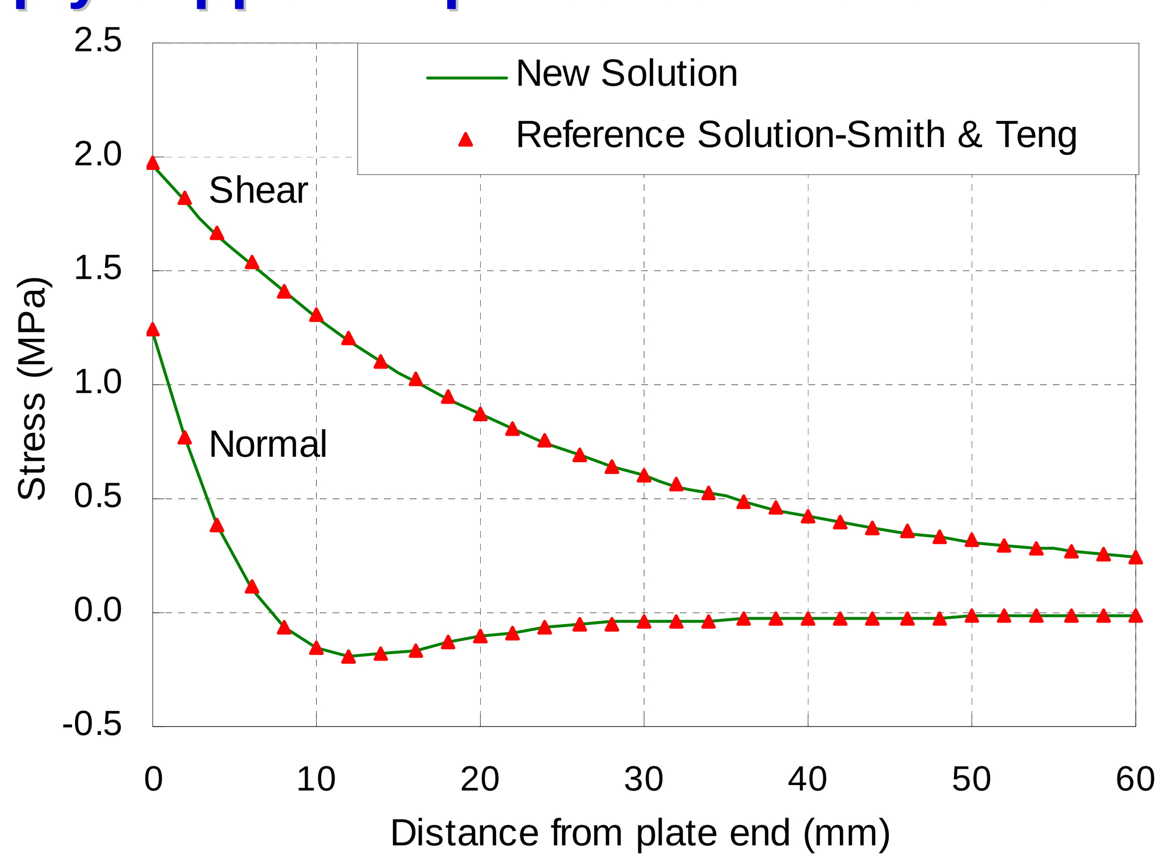
$$C_{2u} = -\frac{E_a}{2\beta^2 t_a}\left(\frac{1}{E_1 I_1}M_{1u} + \frac{1}{E_2 I_2}M_{pu}\right) + \frac{B_{1u} n_1 \lambda^3}{2\beta^2}$$

$V_{Tc}(x)$ = shear force; $Q_e(y)$ = first moment of area of equivalent composite beam section; λ, n_1, n_2, r_1 - r_4, = constants obtained from material & geometrical properties of the beam, adhesive & plate; G_a & t_a = shear modulus & thickness of adhesive.

An Example : Simply supported plated beam under UDL

Data:

Beam : Concrete ;
Plate : GFRP ;
E_1 = 30 GPa ; E_2 = 50 GPa
E_a = 2 GPa ; t_1 = 300 mm
t_2 = 4 mm ; t_a = 2 mm
b_2 = 4 mm ; UDL = 50 kN/ m
a = 300 mm ; L = 3000 mm



Conclusions

1. A novel method based on the superposition principle has been used to obtain a new solution of interfacial shear and normal stresses in beams bonded with a tension face thin plate;
2. The closed-form solution is simpler than all previous solutions;
3. It is applicable to any beam cross sections under any loading arrangement.

References

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