Quantitative Phase Contrast Imaging

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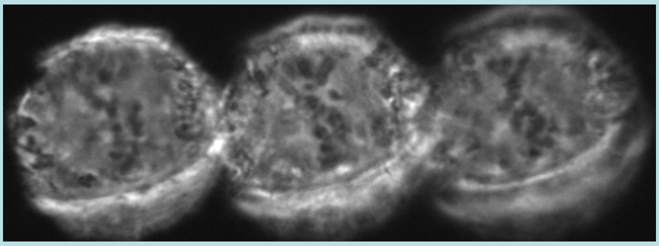
Background

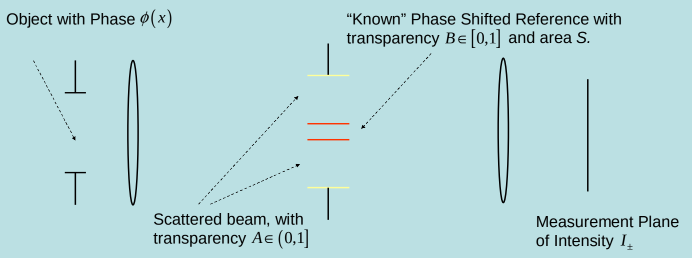
- Applications in astronomy eg. segmented mirrors and biology.
- 1 image is use in conventional phase contrast in this case the benefits of 3 is examined.
- The method presented uses 3 images, one is unaltered and the other two have equal and opposite shifts applied over some finite region.



Experimental example showing cells under going mitosis.

The images are only to illustrate the idea as they are phase contrast images relating to different planes

Model



- 3 measurements with phase shifts $\pm\theta, 0$
- The reference beam is assumed to be unaffected by the scattered beam an therefore can be expressed as

$$R(x) = \mathfrak{F}\left\{\mathfrak{F}\left\{P(-x)\sqrt{I(-x)}\right\}S(\xi)\right\}.$$

Theory

The complex amplitude in the plane of the phase object is defined as

$$\psi(x) = P(x)\sqrt{I(x)}\exp\left(-i\left(\phi(x)\right)\right).$$

Where I defines the intensity, φ the phase and P the objects support.

By applying the phase shift in Fourier space the intensity in measurement plane can be written as

$$I_{\pm}(-x) \cong A^2 I(x)P(x) + \left(A^2 + B^2 - 2AB\cos(\theta)\right)|R(x)|^2 + 2AP(x)\sqrt{I(x)} \times \begin{pmatrix} +\Im\{R(x)\}\left((A - B\cos(\theta))\sin(\alpha(x)+\phi(x)) \mp B\sin(\theta)\cos(\alpha(x)+\phi(x))\right) \\ -\Re\{R(x)\}\left((A - B\cos(\theta))\cos(\alpha(x)+\phi(x)) \pm B\sin(\theta)\sin(\alpha(x)+\phi(x))\right) \end{pmatrix}$$

The sine and cosine object phase can then be calculated from the sum and the difference of the phase shifted intensity with the third used for normalisation. Clearly from there the phase can be calculated within the range $\phi \in (-\pi, \pi]$

$$\sin(\phi(x)) \cong \frac{1}{|R(x)|^2}\left(\frac{\Re\{R(x)\}\Delta I(-x)}{\rho(x)} - \frac{\Im\{R(x)\}\left(\Sigma I(-x) - \mu(x)\right)}{\eta(x)}\right)$$

$$\cos(\phi(x)) \cong \frac{1}{|R(x)|^2}\left(\frac{\Im\{R(x)\}\Delta I(-x)}{\rho(x)} + \frac{\Re\{R(x)\}\left(\Sigma I(-x) - \mu(x)\right)}{\eta(x)}\right).$$

$$\mu(x) = 2\left(P(x)A^2 I(x) + |R(x)|^2\left(A^2 + B^2 - 2AB\cos(\theta)\right)\right),$$

$\eta(x) = 4AP(x)\sqrt{I(x)}\left(B\cos\theta - A\right)$ and $\rho(x) = -4ABP(x)\sqrt{I(x)}\sin\theta.$

Simulations

- Range limited to weak phase objects due to influence from the scattered beam into the reference. Analytic solution from 3 images.

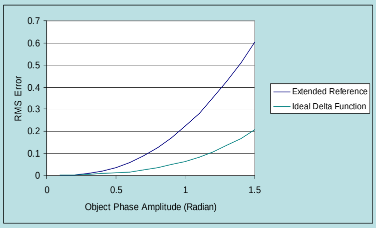


Fig 1. Error between the set and retrieved solution for an increasing object phase amplitude with an extended reference beam compared to a delta function.

- Iteration can be used for refinement provided the phase is small and object is not close to the edge
- Using the equations above an iterative scheme can be constructed

- Comparison between solutions showing the benefit of using more images with an extended reference beam

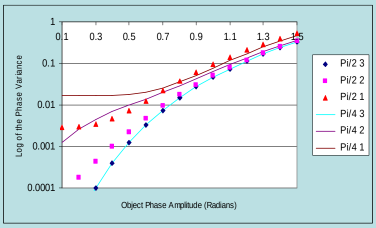


Fig 2. Variance plotted as a function of the object amplitude for difference phase shifts and combinations of images. Pi/2 3 for the 3 image method with a shift, Pi/2 2 uses 2 images and Pi/2 1 for the single-image (i.e. usual phase-contrast method), similarly for the Pi/4 set

- In the previous graph, when using 3 image the solution is independent of the phase shift angle. Clearly in an experiment there are other factors influencing the solution such as noise.

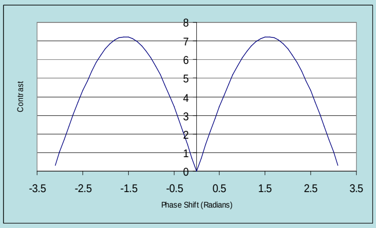


Fig 3. Contrast plotted as a function of phase shift using the 3 image method

- In practice high contrast is desirable for good SNR

Optimum phase shift in this case is $\sim \pm\frac{\pi}{2}$

- In this case the band limit is imposed to implicitly apply continuity and an extended reference beam is used.

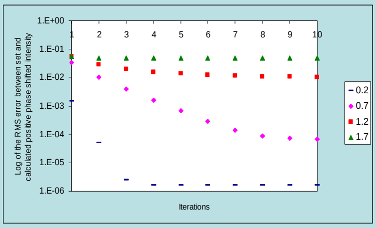


Fig 4. Log of the RMS error is plotted as a function of iterations for varying phase amplitudes (radians).

- In this case the band limit was imposed to implicitly apply continuity and an idealised delta function reference beam was used.

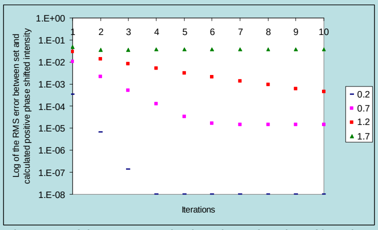


Fig 5. Log of the RMS error is plotted as a function of iterations for varying phase amplitudes (radians).

- In this case there is no band limit is imposed and the reference beam extends over several pixels.

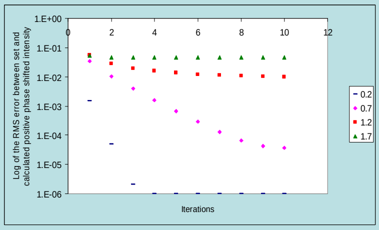


Fig 6. Log of the RMS error is plotted as a function of iterations for varying phase amplitudes (radians).

Conclusions

- Quantitative phase reconstruction with extended phase shifting spot for small phase amplitudes is possible.
- The size of the reference beam is clearly important but as the method is limited to use with weak phase objects the difference is small.
- More images improves the accuracy for weak phase objects but using a single image may be better when imaging larger amplitudes objects when noise is included.
- Large amplitude phase object reconstruction fails even with iteration, as the reference beam is considerably different than expected.
- Including continuity in the iterative scheme does not help improve the dynamic range. The dominant factor influencing the reconstruction is therefore an inability to know the reference beam.