The Edinburgh Research Partnership in Engineering and Mathematics

Joint Research Institute: Signal and Image Processing

Robust Transmit Beamforming Based on Probabilistic Constraint

Huiqin Du, Pei-Jung Chung, Jacek Gondzio, Bernard Mulgrew

Motivation

• Transmit beamforming can enhance SNR performance and channel capacity with perfect channel state information at transmitter (CSIT).

• The accuracy CSIT is typically not available in practice due to the error induced by imperfect (quantized, erroneous, or outdated) channel feedback. The error causes SNR performance degradation.

• The robust design should take errors in CSIT into account to avoid performance degradation.

Problem Formulation

• When the channel matrix is perfectly known, the average SNR at the maximum ratio combining output is

$$SNR = \frac{E_s}{N_0} tr\{\underline{\underline{C}}^H \underline{\underline{H}}^H \underline{\underline{H}}\ \underline{\underline{C}}\}$$

where $E_s = \mathbf{E}[|s|^2]$ is the average signal energy, N_0 is the noise variance, and $\underline{\underline{C}}$ is the beamformer matrix.

• The SNR estimated at transmitter is random because of errors in channel statistics.

$$f(SNR) = \frac{E_s}{N_0} tr\{\underline{\underline{C}}\ \underline{\underline{C}}^H (\hat{\underline{\underline{R}}} + \underline{\underline{E}})\}$$

where the error matrix is denoted by $\underline{\underline{E}}$, the perfect channel covariance matrix is denoted by $\underline{\underline{R}}$ and the estimate channel covariance is defined as $\hat{\underline{\underline{R}}}$

$$\underline{\underline{E}} = \underline{\underline{R}} - \hat{\underline{\underline{R}}}, \quad \underline{\underline{R}} = \mathbf{E}[\underline{\underline{H}}\ \underline{\underline{H}}^H]$$

• Because SNR is random, we consider its expectation

$$\mathbf{E}\ [f(SNR)] = \frac{E_s}{N_0} \mathbf{E}\ [tr\{\underline{\underline{C}}\ \underline{\underline{C}}^H (\hat{\underline{\underline{R}}} + \underline{\underline{E}})\}]$$

and maximize the expected SNR over $\underline{\underline{C}}$.

• Since the worst operational condition is rare, we keep a low outage probability

$$\Pr\{\frac{E_s}{N_0} tr\{\underline{\underline{C}}\ \underline{\underline{C}}^H (\hat{\underline{\underline{R}}} + \underline{\underline{E}})\} \le \gamma_{th}\} \le p_{out}$$

where $\Pr\{A\}$ denotes the probability of the event A.

• The power limitation of the transmit beamformer is formulated as $tr\{\underline{\underline{C}}\ \underline{\underline{C}}^H\} = 1$.

Robust Beamforming Based on Probabilistic -Constrained Optimization

The proposed beamformer maximizes the average SNR and keeps the probability of the worst-case performance at low level under transmit power limitation, that is

$$\max_{\underline{\underline{C}}} \frac{E_s}{N_0} E[tr\{\underline{\underline{C}}\ \underline{\underline{C}}^H (\hat{\underline{\underline{R}}} + \underline{\underline{E}}\}], \quad (1)$$

subject to

$$\Pr\{\frac{E_s}{N_0} tr\{\underline{\underline{C}}\ \underline{\underline{C}}^H (\hat{\underline{\underline{R}}} + \underline{\underline{E}})\} \le r_{th}\} \le p_{out}$$

$$tr\{\underline{\underline{C}}\ \underline{\underline{C}}^H\} = 1$$

where γ_{th} is a SNR threshold, and p_{out} is a pre-specified outage probability.

Assuming the error is complex Gaussian distributed, the probabilistic-constrained optimization problem (1) can be converted into a convex optimization problem.

• Relaxation of Strict Convex Constraint

As the constraint $tr\{\underline{\underline{C}}\ \underline{\underline{C}}^H\} = 1$ is strict, we convert it into an inequality constraint which is easier to satisfy,

$$tr\{\underline{\underline{C}}\ \underline{\underline{C}}^H\} \le 1$$

• Reformulation of Probabilistic Constraint

To simplify the expression in (1), we consider the eigen-decomposition, that is

$$\underline{\underline{C}}\ \underline{\underline{C}}^H = \underline{\underline{U}}_c \underline{\underline{D}}_c \underline{\underline{U}}_c^H \quad \hat{\underline{\underline{R}}} = \hat{\underline{\underline{U}}}_h \hat{\underline{\underline{D}}}_h \hat{\underline{\underline{U}}}_h^H .$$

The diagonal matrix $\underline{\underline{D}}_c = diag(d_1, d_2, \cdots, d_{N_t})$ is the eigenvaules of $\underline{\underline{C}}\underline{\underline{C}}^H$, where $d_1 \ge d_2 \ge \cdots \ge d_{N_t}$.

The corresponding eignevectors are summarized in the unitary matrix $\underline{\underline{U}}_c$. The matrices $\hat{\underline{\underline{D}}}_h = diag(\hat{d}_{h_1}, \hat{d}_{h_2}, \cdots, \hat{d}_{h_{N_t}})$, and $\hat{\underline{\underline{U}}}_h$ are similarly defined.

Under the assumption that the error is complex Gaussian-distributed, i.e. $\underline{\underline{E}} \sim CN(\underline{0}, \sigma_e^2 \underline{\underline{I}})$, the probabilistic constraint can be equivalently converted into a deterministic one

$$F^{-1}(\sqrt{p_{out}})\sigma_e^2 \| \underline{\underline{D}}_c \|_F \ge \gamma_{th} N_0 / (\sqrt{2}\ E_s) - tr\{\underline{\underline{D}}_c \hat{\underline{\underline{D}}}_H\}$$

which is convex when $p_{out} \le 0.25$. $F^{-1}(\cdot)$ is the inverse function of the standard normal distribution, $\| \underline{\underline{D}}_c \|_F$ is the Frobenius norm of $\underline{\underline{D}}_c$.

Simulation

• We consider a single-user MIMO system with $N_t = 4$ transmit antennas and $N_r = 3$ receive antennas.

• The error variance σ_e^2 varies from 0 to 1. The SNR average over 10^4 Monte Carlo trails.

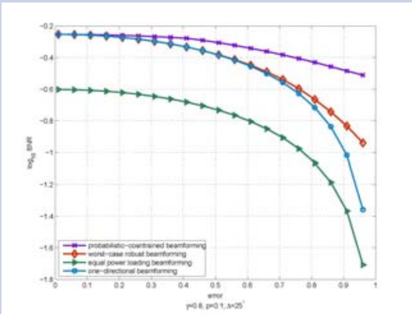

Figure: SNR performance versus error: $\gamma_{th} = 0.8$; $p_{out} = 0.1$; *and the spread angle is* 25^o. *Comparison with one-directional beamformer, equal-power loading beamformer, worst-case beamformer* [1] *and proposed beamformer* [2].

Conclusion

• We propose a novel robust transmit beamforming technique that maximizes the average SNR and keeps the probability of the worst-case performance at a very low level

• The proposed beamformer outperforms several popular robust beamforming algorithms. The improvement are significant for large errors

References

[1] A. Abdel-Samad and A. Gershman, "Robust transmit eigen-beamforming with imperfect knowledge of channel correlations", *IEEE Internatinal Conference on Communication, 2005 (ICC 2005)*, vol. 4, pp.2292-2296, May 2005

[2] H.-Q. Du, P.-J. Chung, J. Gondzio, and B. Mulgrew, "Robust transmit beamforming based on probabilistic constraint", in Processing EUSIPCO, Lausanne, Switzerland, August 2008.