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Parallel Reversible Jump Markov Chain Monte Carlo Computation for Varying-dimension Signal Analysis

Jing Ye
Supervisors: Andrew Wallace, John Thompson

Introduction

LiDAR signal analysis by RJMCMC algorithms based on a Bayesian framework has resulted in greater range accuracy and signal sensitivity, but at larger computational expense. Current parallel MCMC implementations have usually been restricted to a fixed dimension parameter vector. The goal is an efficient parallel solution for the varying-dimension problem.

RJMCMC for LiDAR Signal Analysis

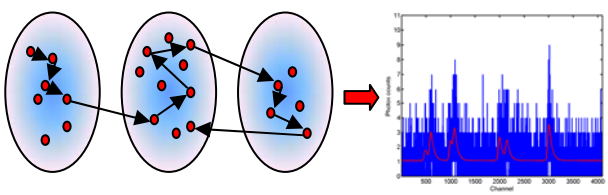
The objective is to calculate the posterior distribution of parameter set:

$$\pi(k,\beta,t_0,B\,|\,Y) \propto L(Y\,|\,k,\beta,t_0,B)\pi(k,\beta,t_0,B)$$

Posterior — Likelihood — Prior

Peak number — Peak amplitudes — Peak positions — Background level

The RJMCMC method allows the Markov chain to jump between multiple parameter subspaces with changing dimensions, and to explore the within-model subspaces using a serial sequence. Hence, it is an attractive method to estimate the posterior distribution of LiDAR signals with unknown numbers of returns.

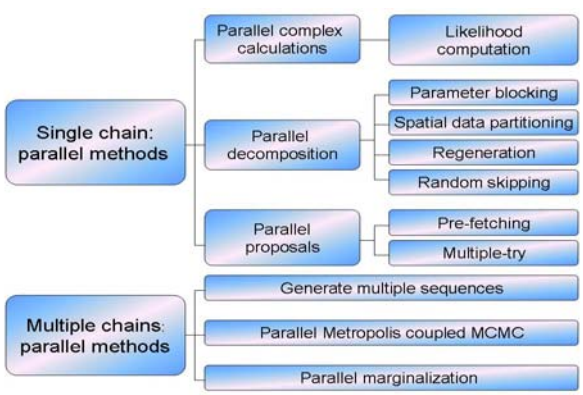


Subspace 1 Subspace 2 Subspace 3

Drawback of long sequence { large computation time; significant memory storage

Parallel MCMC for Fixed-Dimension Problems

Parallel processing improves computational efficiency by distributing the sub-tasks among a group of separate processors. For RJMCMC, parallelization is not only a problem of implementation, but poses statistical challenges. MCMC is sequential by nature and thus is not easily migrated onto a parallel system. Current methods achieve parallelization by making use of the conditional independence of underlying models.



Parallel MCMC for Varying-dimension Problems

One possible solution to the varying-dimension problem is to construct multiple within-model MCMC chains in different parameter subspaces of fixed dimension in parallel. We can improve efficiency by reducing the length of each individual chain in comparison with a serial reversible jump algorithm. The target distribution for a within-modal MCMC chain is:

$$\pi(\theta_k\,|\,k,Y) = \frac{P(k)P(\theta_k\,|\,k)\times L(Y\,|\,k,\theta_k)}{P(k)\times P(Y\,|\,k)} \propto P(\theta_k\,|\,k)\times L(Y\,|\,k,\theta_k)$$

Parameter set — Model indicator — Marginal likelihood

The posterior distribution for a model indicator is then estimated analytically based on the following expressions:

$$\frac{P(k_1\,|\,Y)}{P(k_2\,|\,Y)} = \frac{P(k_1)P(Y\,|\,k_1)}{P(k_2)P(Y\,|\,k_2)} \qquad \frac{P(Y\,|\,k_1)}{P(Y\,|\,k_2)} \rightarrow \text{Bayes Factor}$$

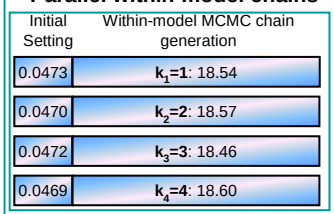
$$P(Y\,|\,k) = \frac{L(Y\,|\,k,\theta_k^*)P(\theta_k^*\,|\,k)}{P(\theta_k^*\,|\,k,Y)}$$

For any chosen point θ_k^*

Experimental Results

To date, we have investigated the similar benchmarking, coal mining disaster problem. This allows comparison with other authors, and is easily re-coded for LiDAR data. It is a non-homogeneous Poisson process, in which the Poisson rate is a step function with unknown number of change points.

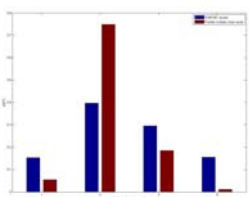
Parallel within-model chains



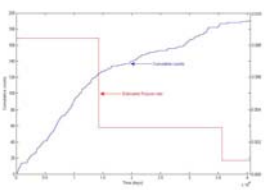
We use four processors to generate four within-model chains each with k_i number of change points. The above figure shows the timing results for 40000 samples measured in seconds.

This is the final estimation results with two change points. The Poisson rates are set to be the posterior distribution mode.

$P(k\,|\,Y)$ Calculation



Comparison of *P(k|Y)* by the RJMCMC chain and the Bayes factor.



Conclusion

This first approach gives almost linear speed-up, but the serial and parallel algorithms differ in the way they explore the parameter space, and in the posterior estimates. We aim to develop optional parallel decompositions and to determine the validity of these estimates and of the convergence criteria used to stop the chain. Other improvements including optimal compilation, vectorized code, the use of FPGAs and algorithmic changes should achieve real-time performance.