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Bayesian Filtering, Smoothing and Distributed Data Fusion for Joint Detection and Tracking

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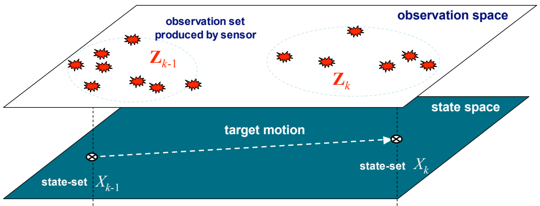
Joint Target-Detection and Tracking (JoTT)



Bayes filter

$$\cdots \rightarrow f_{k-1}(\mathbf{X}_{k-1}|\mathbf{Z}_{1:k-1}) \xrightarrow{\text{prediction}} f_{k|k-1}(\mathbf{X}_k|\mathbf{Z}_{1:k-1}) \xrightarrow{\text{data-update}} f_k(\mathbf{X}_k|\mathbf{Z}_{1:k}) \rightarrow \cdots$$

new observation set

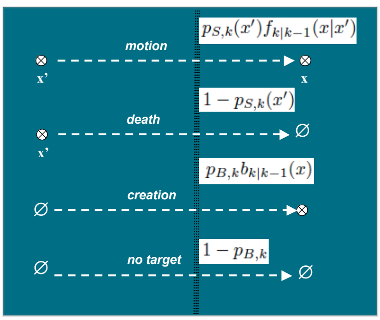
$$\int f_{k|k-1}(\mathbf{X}_k|\mathbf{X}_{k-1})\, f_{k-1}(\mathbf{X}_{k-1}|\mathbf{Z}_{1:k-1})\,\delta\mathbf{X}_{k-1}$$

$$K^{-1} f_{k|k-1}(\mathbf{X}_k|\mathbf{Z}_{1:k-1})\, g_k(\mathbf{Z}_k|\mathbf{X}_k)$$

Bernoulli Markov Transition — Random Finite Set Likelihood

JoTT Markov Motion Model

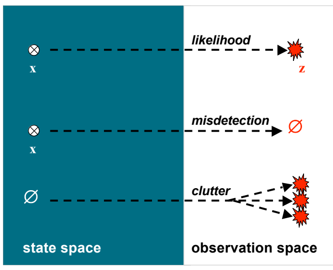
$X_{k-1} \dashrightarrow X_k$



$f_{k|k-1}(X_k|X_{k-1})$

Joint Target-Detection and Tracking transition density

Random Finite Set Observation Model



Random set of observations — Bernoulli object state

$g_k(Z_k|X_k)$

Random-set observation likelihood

Clutter intensity — Missed detection term — probability of object detection — Single object likelihood

$$g_k(Z_k|X_k) = \begin{cases} \kappa_k(Z_k), & X_k = \phi \\ \kappa_k(Z_k)\left((1-p_{D,k}(x_k)) + p_{D,k}(x_k)\sum_{z\in Z_k}\left(L_z(x_k)\frac{\kappa_k(Z_k-\{z\})}{\kappa_k(Z_k)}\right)\right), & X_k = \{x_k\}, \end{cases}$$

Joint Target-Detection and Tracking Filter

- **Bernoulli State Bayes Prediction**

Bernoulli Markov transition

$$f_k^{1:k-1}(X_k|Z_{1:k-1}) = \int f_{k|k-1}(X_k|X_{k-1})\, f_{k-1}^{1:k-1}(X_{k-1}|Z_{1:k-1})\,\delta X_{k-1},$$

Predicted Bernoulli density — posterior at time k-1 — Bernoulli process integral

- **Bernoulli State Bayes Update**

Bayes updated Bernoulli density — Likelihood for Random Set Observations — Predicted Bernoulli density

$$f_k^{1:k}(X_k|Z_{1:k}) = \frac{g_k(Z_k|X_k)\, f_k^{1:k-1}(X_k|Z_{1:k-1})}{\int g_k(Z_k|X)\, f_k^{1:k-1}(X|Z_{1:k-1})\,\delta X},$$

Gaussian Mixture JoTT Filter

- A closed-form solution to the JoTT filter recursion exists for linear Gaussian multi-target model
- Probability of target existence recursion has analytic solution
- Gaussian mixture prior density results in Gaussian mixture posterior density:

Gaussian mixture JoTT filter

$$\cdots \rightarrow p_{k-1}^{1:k-1} \rightarrow p_k^{1:k-1} \rightarrow p_k^{1:k} \rightarrow \cdots$$

$$\cdots \rightarrow f_{k-1}^{1:k-1}(x'|Z_{1:k-1}) \rightarrow f_k^{1:k-1}(x|Z_{1:k-1}) \rightarrow f_k^{1:k}(x_k|Z_{1:k}) \rightarrow \cdots$$

$$\cdots \rightarrow \{w_{k-1}^{(i)}, \mathbf{m}_{k-1}^{(i)}, P_{k-1}^{(i)}\}_{i=1}^{J_{k-1}} \rightarrow \{w_{k|k-1}^{(i)}, \mathbf{m}_{k|k-1}^{(i)}, P_{k|k-1}^{(i)}\}_{i=1}^{J_{k|k-1}} \rightarrow \{w_k^{(i)}, \mathbf{m}_k^{(i)}, P_k^{(i)}\}_{i=1}^{J_k} \rightarrow \cdots$$

JoTT Forward-Backward Smoother

Bayes smoothed Bernoulli density — Forward updated posterior at time k — Bernoulli Markov transition — Smoothed density at time k+1

$$f_k^{1:N}(X_k|Z_{1:N}) = f_k^{1:k}(X_k|Z_{1:k}) \int \frac{f_{k+1|k}(X_{k+1}|X_k)\, f_{k+1}^{1:N}(X_{k+1}|Z_{1:N})}{f_{k+1}^{1:k}(X_{k+1}|Z_{1:k})}\,\delta X_{k+1},$$

Predicted density to time k+1

The smoothed probability of target existence and track density are:

$$p_k^{1:N} = 1 - \left((1-p_k^{1:k})\left(\frac{(1-p_{B,k})(1-p_{k+1}^{1:N})}{(1-p_{k+1}^{1:k})} + \int \frac{p_{B,k}b_{k+1|k}(x')p_{k+1}^{1:N}f_{k+1}^{1:N}(x'|Z_{1:N})}{p_{k+1}^{1:k}f_{k+1}^{1:k}(x'|Z_{1:k})}dx'\right)\right)$$

$$f_k^{1:N}(x|Z_{1:N}) = \frac{p_k^{1:k}f_k^{1:k}(x|Z_{1:k})}{p_k^{1:N}}\left(\frac{(1-p_{S,k}(x))(1-p_{k+1}^{1:N})}{(1-p_{k+1}^{1:k})} + \int \frac{p_{S,k}(x)f_{k+1|k}(x'|x)p_{k+1}^{1:N}f_{k+1}^{1:N}(x'|Z_{1:N})}{p_{k+1}^{1:k}f_{k+1}^{1:k}(x'|Z_{1:k})}dx'\right)$$

JoTT Two-Filter Smoother

- The backward-prediction, backward-update and smoothed estimate are

Backward predicted density — Backward posterior update at time k+1 — Markov transition — Artificial distribution

$$f_k^{k+1:N}(X_k|Z_{k+1:N}) = \int f_{k+1}^{k+1:N}(X_{k+1}|Z_{k+1:N})\frac{f_{k+1|k}(X_{k+1}|X_k)\gamma_k(X_k)}{\gamma_{k+1}(X_{k+1})}\delta X_{k+1}$$

$$f_k^{k:N}(X|Z_{k:N}) = \frac{g_k(Z_k|X)f_k^{k+1:N}(X|Z_{k+1:N})}{\int g_k(Z_k|X)f_k^{k+1:N}(X|Z_{k+1:N})\delta X.}$$

$$f_k^{k:N}(X_k|Z_{1:N}) \propto \frac{f_k^{1:k-1}(X_k|Z_{1:k-1})f_k^{k:N}(X_k|Z_{k:N})}{\gamma_k(X_k)}$$

Smoothed density at time k — Forward prediction to time k — Backward update at time k

Distributed Data Fusion

- Two multi-object posteriors can be combined to form a fused estimate with information from both

$$f_w(X|Z_0^k, Z_1^k) = \frac{f_0(X|Z_0^k)^{(1-w)}f_1(X|Z_1^k)^w}{\int f_0(Y|Z_0^k)^{(1-w)}f_1(Y|Z_1^k)^w\delta Y}$$

- For Bernoulli posteriors, this can be computed explicitly

$$p_w = 1 - \frac{1}{K}(1-p_0)^{(1-w)}(1-p_1)^w$$

$$f_w(x|Z_0^k, Z_1^k) = \frac{1}{p_wK}p_0^{(1-w)}p_1^w\int f_0(x|Z_0^k)^{(1-w)}f_1(x|Z_1^k)^w dx$$

- The optimal value of 'w' can be found with the Chernoff Information

$$C(f_0,f_1) = -\min_{1\le w\le 1}\log\left(\int f_0(Y|Z_0^k)^{(1-w)}f_1(Y|Z_1^k)^w\delta Y\right)$$